100

Please note that there are 10 questions

1) Lafayette Savings Bank offers an account that pays 4% interest compounded four times a year. They plan to change to monthly compounding. What interest rate, compounded monthly, should they offer in order to have the same annual effective rate of interest? (10 pts)

$$\left(1+\frac{.04}{4}\right)^4 = \left(1+\frac{i}{12}\right)^{12}$$

✓

$$\left(\left(1+\frac{.04}{4}\right)^{\frac{4}{12}} - 1\right)12 = i = .039867403$$

$i =$ 3.99.1

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2) You borrow \$2000 at the beginning of year 1 at 7% annual effective interest. You pay \$P at the end of year 1 , \$500 at the end of year 2, and \$1000 at the end of year 3 , after which you owe nothing. Find P.
(10 pts)

$$2000(1.07)^3 = P(1.07)^2 + 500(1.07) + 1000$$

$P = \$799.27$

3) From January 1, 2005 to December 31, 2010, First Bank paid 1% interest, compounded monthly. On January 1, 2011, they raised their rate to 3% interest, compounded monthly. I deposited $200 at the end of each month beginning in January, 2005. How much will I have in my account immediately after my deposit on December 31, 2014.
(10 pts)

$$\left(200\frac{\left(1+\frac{.01}{12}\right)^{12\cdot 6}-1}{.01/12}\right)\left(1+\frac{.03}{12}\right)^{12\cdot 4}+200\frac{\left(1+\frac{.03}{12}\right)^{48}-1}{.03/12}$$

$$16{,}723.23904 \quad + \quad 10{,}186.24168$$

$$= \quad 26{,}909.48072$$

$$\downarrow$$

$$\boxed{\$\ 26{,}909.48}$$

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4) Over a 10 year period an account earned 2% annual effective interest for the first two years, 4% annual effective discount for the next three years, and 5% annual effective force of interest for the last 5 years. What was the annual effective rate of return on this account? (10 pts)

$$(1.02)^2\left(1+\frac{.04}{1-.04}\right)^3\left(e^{.05}\right)^5 = (1+i)^{10}$$

$$1.042068792 = 1+i$$

$$i = 4.207\%$$ ✓

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5) On January 1 you won a prize that pays $100,000 at the end of each year for n years. Find n given that the present value of your prize at the beginning of the first year at 4% interest was $1,562,207.
(10 pts)

$$100{,}000\,\frac{(1.04)^n - 1}{.04} = 1{,}562{,}207(1.04)^n$$

$$(1.04)^n - 1 = 0.6248828(1.04)^n$$

$$(1.04)^n(1 - 0.6248828) = 1$$

$$1.04^n = 2.665833505$$

$$\frac{\log}{\log 1.04} = n$$

$n = 24.99997297$ years

$25 = n$

6) You borrow $500,000 to buy a house at 4% interest, compounded monthly, with a 30 year mortgage. What is your monthly payment (paid at the end of each month)? (10 pts)

$$\frac{500{,}000\left(1+\frac{.04}{12}\right)^{360}}{\frac{\left(1+\frac{.04}{12}\right)^{360}-1}{.04/12}} = P$$

$P = \$2{,}387.076427$

$\$2{,}387.08$

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7) Let the data be as in problem 6). Assume also that you have actually been paying $4,000 each month, which is somewhat more than required. After paying on the house for ten years, i.e. immediately after the 120th payment, interest rates have dropped to 3%. You decide to refinance. How much will you need to borrow to pay off your original loan? (10 pts)

Amount Left on orig. $= 500000\left(1+\frac{.04}{12}\right)^{120} - 4000\left(\frac{\left(1+\frac{.04}{12}\right)^{120}-1}{.04/12}\right)$

$= 745416.3409 - 588999.2183 = \$156417.1226$

Amount that will be paid over 20 years w/ comp. mo. interest $= 156417.1226\left(1+\frac{.03}{12}\right)^{12\cdot 20} = \$284797.2573$

8) What price should you pay for a $10,000 redemption value, 20 year bond which has $200 coupons, paid three times a year, assuming that you want a 3% yield, compounded three times a year? (10 pts)

$$\left(200\left(\frac{\left(1+\frac{.03}{3}\right)^{60}-1}{.03/3}\right)+10{,}000\right)\left(1+\frac{.03}{3}\right)^{-60}$$

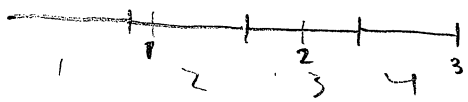
$$= \$14{,}495.50$$

(9) The bond in question 8) is sold after five years, immediately after the payment of the coupon, to an investor wanting a 4% yield, compounded quarterly? What should the selling price of the bond be? (10 pts)

$$\left(\frac{\left(\left(1+\frac{.04}{4}\right)^{\frac{4}{3}}\right)^{15\cdot 3}-1}{\left(1+\frac{.04}{4}\right)^{\frac{4}{3}}-1}\left(200\right)+10{,}000 \right)\left(1+\frac{.04}{4}\right)^{-60}$$

$$= 12{,}236.55665$$

$$\hookrightarrow \boxed{\$12{,}236.56}$$



10) I invest $100 at the **beginning** of each month in an account that pays 4% interest per year, **compounded daily**. What is my accumulation at the end of the 10th year?
(10 pts)

$$\left(1+\frac{.04}{365}\right)^{\frac{365}{12}}(100)\left(\frac{\left(\left(1+\frac{.04}{365}\right)^{\frac{365}{12}}\right)^{120}-1}{\left(1+\frac{.04}{365}\right)^{\frac{365}{12}}-1}\right) = \$14,779.17$$